Gov 2002 - Causal Inference II: Instrumental Variables

Matthew Blackwell Arthur Spirling

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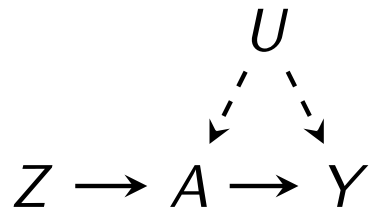
- But if you can identify some exogenous sources of variation that drive the treatment, even if the treatment was not randomly assigned, you may be able to make headway.
- The basic idea behind instrumental variables is that we have a treatment with unmeasured confounding, but that we have another variable, called the instrument, that affects the treatment, but not the outcome, and thus give us that exogenous variation.

Basic IV setup with DAGs



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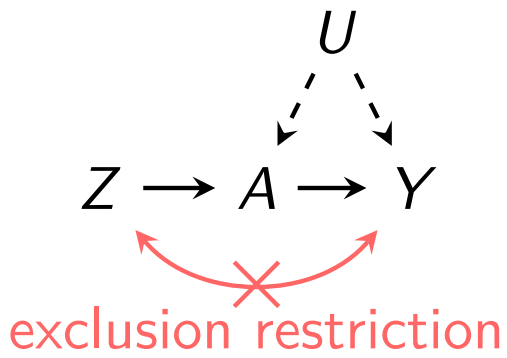

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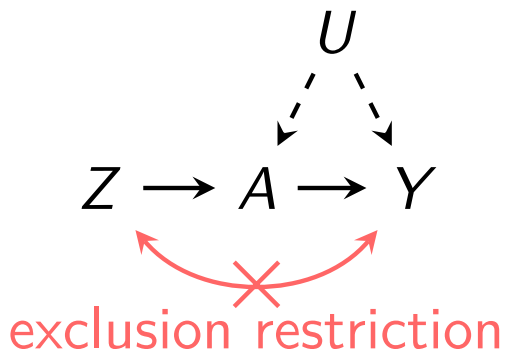

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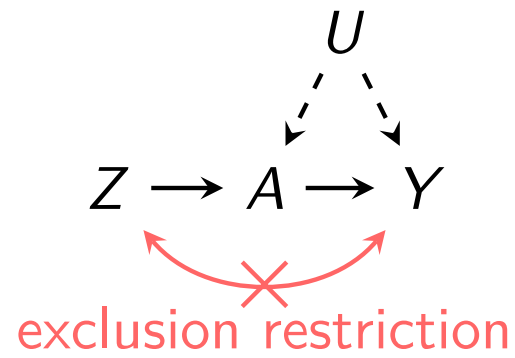

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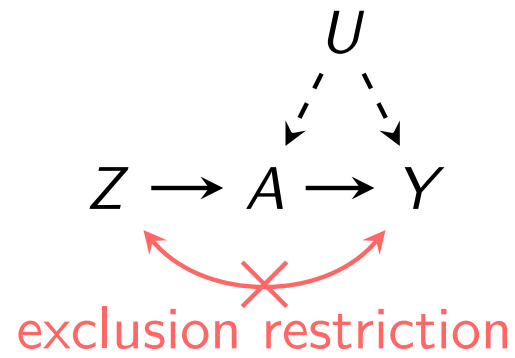

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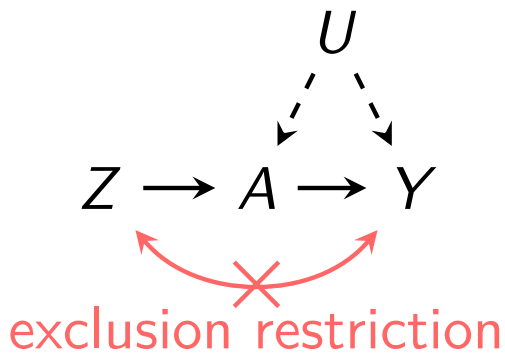

- Z is the instrument, A is the treatment, and U is the unmeasured confounder
- Exclusion restriction
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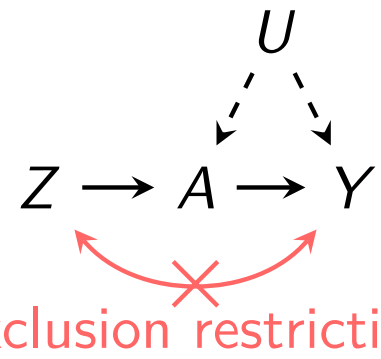
- First-stage relationship: Z affects A

An IV is only as good as its assumptions



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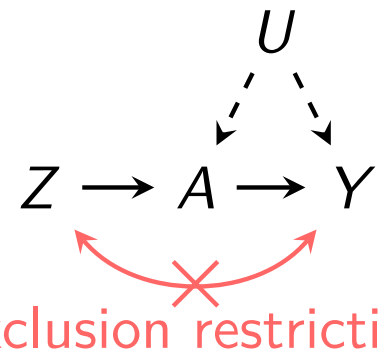

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- We will see that even if all of the untestable assumptions are met, the IV approach estimates a "local" ATE. That is, local to this particular case/instrument.

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- Acharya, Blackwell, Sen (2014): cotton suitability as IV for proportion slave in 1860 (outcome is white attitudes today)

IV with constant effects

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- But $\text{cov}(\gamma U_i + \eta_i, A_i) \neq 0$ because U is a common cause of A and Y.

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$$\begin{aligned}\text{cov}(Y_i, Z_i) &= \text{cov}(\alpha + \tau A_i + \gamma U_i + \eta_i, Z_i) \\ &= \text{cov}(\alpha, Z_i) + \text{cov}(\tau A_i, Z_i) + \text{cov}(\gamma U_i + \eta_i, Z_i)\end{aligned}$$

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&= 0 + \tau \text{cov}(A_i, Z_i) + 0
\end{aligned}$$

IV estimator with constant effects

$$Y_i = \alpha + \tau A_i + \gamma U_i + \eta_i$$

- With this in hand, we can formulate an expression for the average treatment effect here:

$$\tau = \frac{\text{Cov}(Y_i, Z_i)}{\text{Cov}(A_i, Z_i)} = \frac{\text{Cov}(Y_i, Z_i)/V[Z_i]}{\text{Cov}(A_i, Z_i)/V[Z_i]}$$

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- What happens with a weak first stage?

Wald Estimator

- With a binary instrument, there is a simple estimator based on this formulation called the Wald estimator. It is easy to show that:

$$\tau = \frac{\text{Cov}(Y_i, Z_i)}{\text{Cov}(A_i, Z_i)} = \frac{E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]}{E[A_i|Z_i = 1] - E[A_i|Z_i = 0]}$$

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- Intuitively, the effects of Z_i on Y_i divided by the effect of Z_i on A_i

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- . . . but A_i is **endogenous**: $E[A_i\varepsilon_i] \neq 0$

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- Red value in the brackets is the population fitted value of the treatment, $E[A_i|X_i, Z_i]$
- Because Z_i and X_i are uncorrelated with ν_i and ε_i, then this fitted value is also independent of ε_i^*.
- Thus, the population regression coefficient of a Y_i on $[X_i'\alpha + \gamma Z_i]$ is the average treatment effect, τ.

Two-stage least squares

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- Key question: is $\hat{A}_i$ uncorrelated with the error?
- $\hat{A}_i$ is just a function of X_i and Z_i so it is uncorrelated with ε_i.
- We also know that $\hat{A}_i$ is uncorrelated with $(A_i - \hat{A}_i)$?

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  1. Run regression of treatment on covariates and instrument
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- Note that this isn't how we actually estimate 2SLS because the standard errors are all wrong.
- Computer wants to calculate the standard errors based on ε_i^*, but what we really want is the standard errors based on ε_i.

Nunn & Wantchekon IV example

TABLE 5—IV ESTIMATES OF THE EFFECT OF THE SLAVE TRADE ON TRUST

	Trust of relatives (1)	Trust of neighbors (2)	Trust of local council (3)	Intragroup trust (4)	Intergroup trust (5)
Second stage: Dependent variable is an individual's trust					
ln (1 + exports/area)	−0.190***	−0.245***	−0.221***	−0.251***	−0.174**
	(0.067)	(0.070)	(0.060)	(0.088)	(0.080)
Hausman test (p-value)	0.88	0.53	0.09	0.44	0.41
R^2	0.13	0.16	0.20	0.15	0.12
First stage: Dependent variable is ln (1 + exports/area)					
Historical distance of ethnic group from coast	−0.0014***	−0.0014***	−0.0014***	−0.0014***	−0.0014***
	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0003)
Colonial population density	Yes	Yes	Yes	Yes	Yes
Ethnicity-level colonial controls	Yes	Yes	Yes	Yes	Yes
Individual controls	Yes	Yes	Yes	Yes	Yes
District controls	Yes	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes	Yes
Number of observations	16,709	16,679	15,905	16,636	16,473
Number of clusters	147 / 1,187	147 / 1,187	146 / 1,194	147 / 1,186	147 / 1,184
F-stat of excl. instrument	26.9	26.8	27.4	27.1	27.0
R^2	0.81	0.81	0.81	0.81	0.81

Notes: The table reports IV estimates. The top panel reports the second-stage estimates, and the bottom panel reports first-stage estimates. Standard errors are adjusted for two-way clustering at the ethnicity and district levels. The individual controls, district controls, ethnicity-level colonial controls, and colonial population density measures are described in Table 3. The null hypothesis of the Hausman test is that the OLS estimates are consistent.

***Significant at the 1 percent level.

**Significant at the 5 percent level.

*Significant at the 10 percent level.

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- The structural model, then is:

$$Y_i = X_i'\beta + \varepsilon_i$$

- Z_i will be a vector of l exogenous variables that includes any exogenous variables in X_i plus any instruments. Key assumption:

$$E[Z_i\varepsilon_i] = 0$$

Nasty Matrix Algebra

- Useful quantities:

$$\Pi = (E[Z_i Z_i'])^{-1} E[Z_i X_i'] \qquad \text{(projection matrix)}$$

$$V_i = \Pi' Z_i \qquad \text{(fitted values)}$$

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$$V_i = \Pi' Z_i \qquad \text{(fitted values)}$$

- To derive the 2SLS estimator, take the fitted values, $\Pi' Z_i$ and multiply both sides of the outcome equation by them:

$$Y_i = X_i' \beta + \varepsilon_i$$

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- This is how R/Stata estimates the 2SLS parameters

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- Consistent because $n^{-1}\sum_i V_i\varepsilon_i \xrightarrow{p} E[V_i\varepsilon_i] = 0$.

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- Replace with the sample quantities to get estimates:

$$\widehat{\text{var}}(\hat{\beta}) = (V'V)^{-1} \left(\sum_i \hat{u}_i^2 V_i V_i'\right) (V'V)^{-1}$$

where $\hat{u}_i = Y_i - X_i' \hat{\beta}$

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- Is it plausible to find more than one instrument?

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- These overidentification tests depend heavily on the constant effects assumption
- Once we move away from constant effects, we no longer can generally pool multiple instruments together in this way.

Reading

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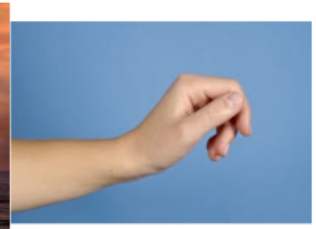

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- Identifies the **intent-to-treat (ITT) effect**:

$$E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0] = E[Y_i(A_i(1), 1) - Y_i(A_i(0), 0)]$$

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The linear model with heterogeneous effects

- Rewriting the usual consistency assumption gives us a linear model with heterogeneous effects (we have seen this before in randomized experiments):

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- Here, we have $\alpha_0 = E[Y_i(0)]$ and $\tau_i = Y_i(1) - Y_i(0)$.

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- Otherwise, what would we be doing? The instrument wouldn't affect anything.

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- Note if this holds in the opposite direction $A_i(1) - A_i(0) \leq 0$, we can always rescale A_i to make the assumption hold.

Monotonicity means no defiers

- This is sometimes called "no defiers". It turns out that with a binary treatment and a binary instrument, we can group units into four categories:

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- The monotonicity assumption remove the possibility of there being defiers in the population.
- Anyone with $A_i = 1$ when $Z_i = 0$ must be an always-taker and anyone with $A_i = 0$ when $Z_i = 1$ must be a never-taker.

Local Average Treatment Effect (LATE)

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- This is is the ATE among the compliers: those that take the treatment when encouraged to do so.
- That is, the LATE theorem, states that:

$$\frac{E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]}{E[A_i|Z_i = 1] - E[A_i|Z_i = 0]} = E[Y_i(1) - Y_i(0)|A_i(1) > A_i(0)]$$

Local Average Treatment Effect (LATE)

- Under these four assumptions, the Wald estimator is equal what we call Local average treatment effect (LATE) or the complier average treatment effect (CATE).
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- This fact was a massive intellectual jump in our understanding of IV.

Proof of the LATE theorem

- Under the exclusion restriction and randomization,

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- The third equality comes from monotonicity: with this assumption, $A_i(1) < A_i(0)$ never occurs.

Proof (continued)

$$E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0] = E[Y_i(1) - Y_i(0)|A_i(1) > A_i(0)] \Pr[A_i(1) > A_i(0)]$$

- We can use the same argument for the denominator:

$$\begin{aligned} E[A_i|Z_i = 1] - E[A_i|Z_i = 0] &= E[A_i(1) - A_i(0)] \\ &= \Pr[A_i(1) > A_i(0)] \end{aligned}$$

- Dividing these two expressions through gives the LATE.

Reading

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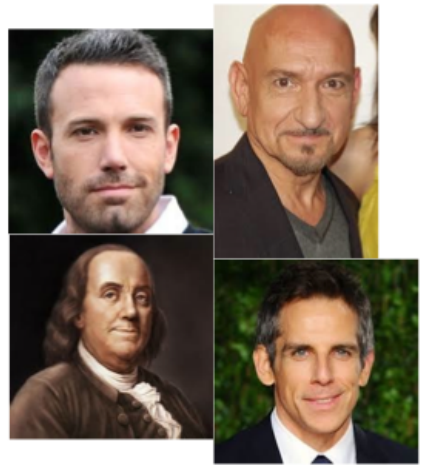

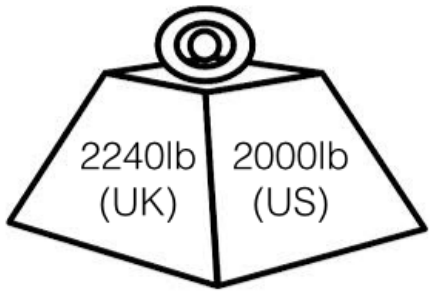

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- Furthermore, since the complier group depends on the instrument, an IV estimate with one instrument will generally estimate a different quantity than an IV estimate of the *same effect* with a different instrument.
- 2SLS "cheats" by assuming that the effect is constant, so it is the same for compliers and non-compliers.

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- Maybe this is because only those treated actually get pills or only they are invited to the job training location.

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(law of iterated expectations + binary treatment)

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- Noting that $\Pr[A_i = 1|Z_i = 0] = 0$, then the Wald estimator is just the ATT:

$$\frac{E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]}{\Pr[A_i = 1|Z_i = 1]} = E[Y_i(1) - Y_i(0)|A_i = 1]$$

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- Thus, under the additional assumption of one-sided compliance, we can estimate the ATT using the usual IV approach .
- The ATT is a combination of the LATE and the ATE for the always-takers. If we remove the possibility of the always takers, then anyone who actually takes the treatment is a complier.
- It's also easy to see that if we switch the direction of one-sided compliance, then we can esimate the average treatment effect for the controls.

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- Nunn & Wantchekon (2011): use distance to coast as an instrument for Africans, use distance to the coast in an Asian sample as falsification test.

Nunn & Wantchekon falsification test

TABLE 7—REDUCED FORM RELATIONSHIP BETWEEN THE DISTANCE FROM THE COAST AND TRUST WITHIN AFRICA AND ASIA

	Trust of local government council			
	Afrobarometer sample		Asiabarometer sample	
	(1)	(2)	(3)	(4)
Distance from the coast	0.00039***	0.00031***	−0.00001	0.00001
	(0.00009)	(0.00008)	(0.00010)	(0.00009)
Country fixed effects	Yes	Yes	Yes	Yes
Individual controls	No	Yes	No	Yes
Number of observations	19,913	19,913	5,409	5,409
Number of clusters	185	185	62	62
R^2	0.16	0.18	0.19	0.22

Notes: The table reports OLS estimates. The unit of observation is an individual. The dependent variable in the Asiabarometer sample is the respondent's answer to the question: "How much do you trust your local government?" The categories for the answers are the same in the Asiabarometer as in the Afrobarometer. Standard errors are clustered at the ethnicity level in the Afrobarometer regressions and at the location (city) level in the Asiabarometer and the WVS samples. The individual controls are for age, age squared, a gender indicator, education fixed effects, and religion fixed effects.

***Significant at the 1 percent level.

**Significant at the 5 percent level.

*Significant at the 10 percent level.

Size, characteristics of the compliers

- While we cannot identify who is a complier and who is not a complier in general, we can estimate the size of the complier group:

$$\Pr[A_i(1) > A_i(0)] = E[A_i(1) - A_i(0)] = E[A_i|Z_i = 1] - E[A_i|Z_i = 0]$$

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- Angrist and Pischke describe ways to calculate the difference between the compliers and overall population in terms of binary covariates.

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- While we cannot identify who is a complier and who is not a complier in general, we can estimate the size of the complier group:

$$\Pr[A_i(1) > A_i(0)] = E[A_i(1) - A_i(0)] = E[A_i|Z_i = 1] - E[A_i|Z_i = 0]$$

- Angrist and Pischke describe ways to calculate the difference between the compliers and overall population in terms of binary covariates.
- Abadie (2003) shows how to calculate the mean of any covariate in the complier group.

Multiple instruments

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- Thus, if we had two instrument, then there would be two different LATEs, ρ_1 and ρ_2 for instruments Z_{1i} and Z_{2i}. We might try to use 2SLS to estimate an overall effect with these instruments with following first stage:

$$\hat{A}_i = \pi_1 Z_{1i} + \pi_2 Z_{2i}.$$

2SLS as weighted average

- In Angrist and Pischke, they show that the 2SLS estimator using these two instruments is a weighted sum of the two component LATEs:

$$\rho_{2SLS} = \psi\rho_1 + (1 - \psi)\rho_2,$$

where the weights are:

$$\psi = \frac{\pi_1 \mathrm{Cov}(A_i, Z_{1i})}{\pi_1 \mathrm{Cov}(A_i, Z_{1i}) + \pi_2 \mathrm{Cov}(A_i, Z_{2i})}$$

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- Thus, the 2SLS estimate is a weighted average of causal effects for each instrument, where the weights are related to the strenght of prediction for each of the first stage effects of the instruments.

Covariates and heterogeneous effects

- It might be the case that the above assumptions only hold conditional on some covariates, X_i. That is, instead of randomization, we might have conditional ignorability:

$$[\{Y_i(a,z), \forall a, z\}, A_i(1), A_i(0)] \perp\!\!\!\perp Z_i | X_i$$

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- Abadie (2003) shows how to estimate the overall LATE using a weighting approach based on a "propensity score" for the instrument.