Gov 50: 12. Linear Regression (II)

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Fall 2018

1/ Today's agenda

Logistics

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  - Extra credit worth a good amount of post-curve grade (3%)

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- Last time: how to use **linear regression** to predict outcomes using another variable.
- Now: assess model fit and use more than 1 variable to predict.

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- Linear regression often used to do these predictions, but how well does our model predict the data?

2/ Model fit

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Name	Description
`year`	midterm election year
`president`	name of president
`party`	Democrat or Republican
`approval`	Gallup approval rating at midterms
`seat.change`	change in the number of House seat’s for the president’s party
`rdi.change`	change in real disposable income over the year before midterms

Loading the data

```
midterms <- read.csv("data/midterms.csv")
head(midterms)
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```
##   year  president party approval seat.change
## 1 1946     Truman     D       33         -55
## 2 1950     Truman     D       39         -29
## 3 1954 Eisenhower     R       61          -4
## 4 1958 Eisenhower     R       57         -47
## 5 1962    Kennedy     D       61          -4
## 6 1966    Johnson     D       44         -47
##   rdi.change
## 1         NA
## 2        8.0
## 3        0.2
## 4       -0.8
## 5        4.1
## 6        3.2
```

Fitting the approval model

```r
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```
##
## Call:
## lm(formula = seat.change ~ approval, data = midterms)
##
## Coefficients:
## (Intercept)     approval
##      -96.84         1.42
```

Fitting the income model

```r
fit.rdi <- lm(seat.change ~ rdi.change, data = midterms)
fit.rdi
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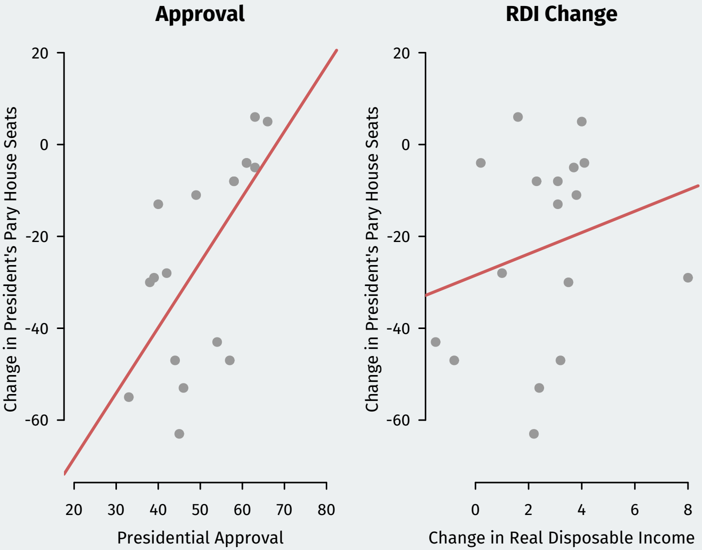
```

```
##
## Call:
## lm(formula = seat.change ~ rdi.change, data = midterms)
##
## Coefficients:
## (Intercept)   rdi.change
##      -28.48         2.33
```

Comparing models



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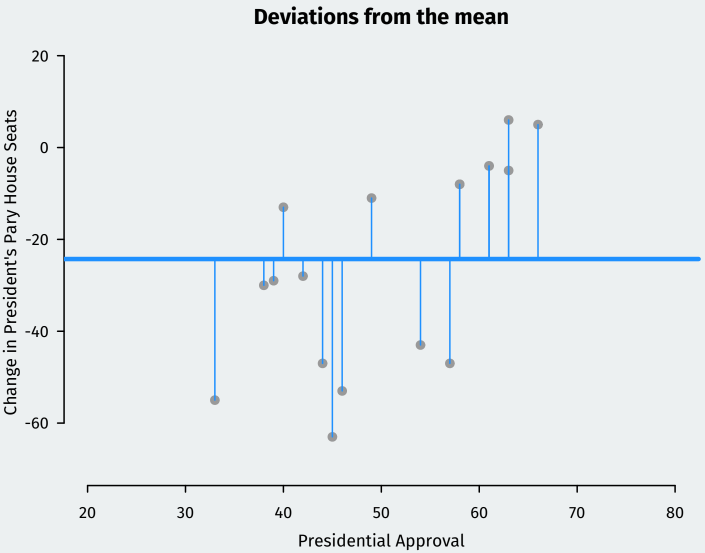
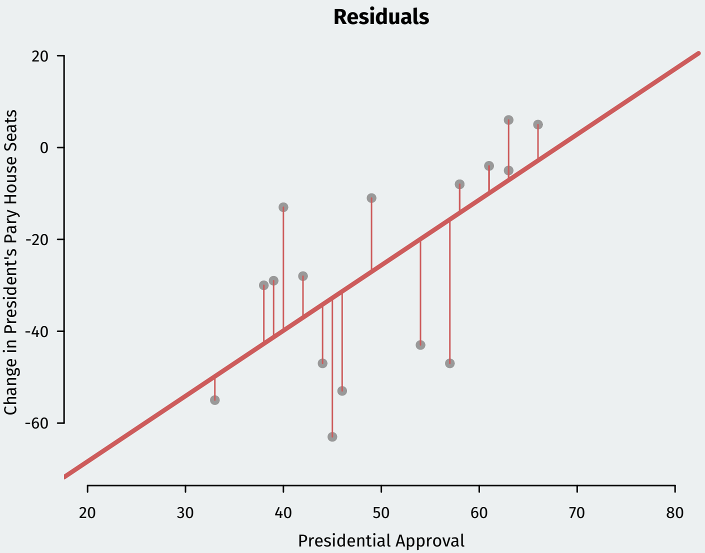
- Roughly: proportion of the variation in Y_i "explained by" X_i

Total SS vs SSR



Total SS vs SSR



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fit.rdi.sum <- summary(fit.rdi)
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```
## [1] 0.0544
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- Which does a better job predicting midterm election outcomes?

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- Very good model fit: $R^2 \approx 0.95$

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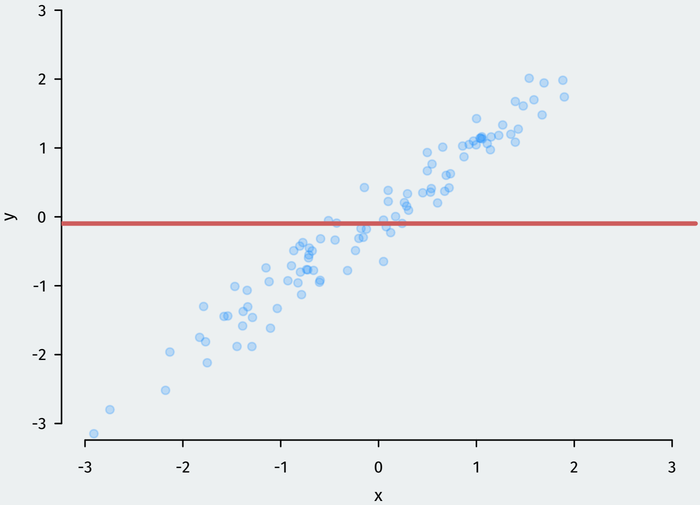

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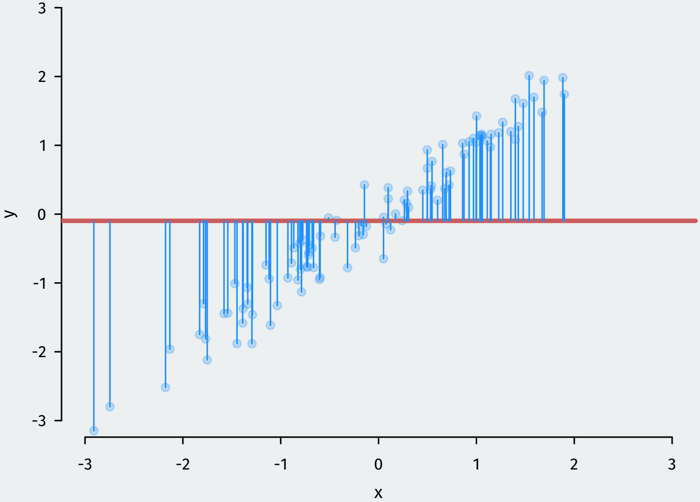

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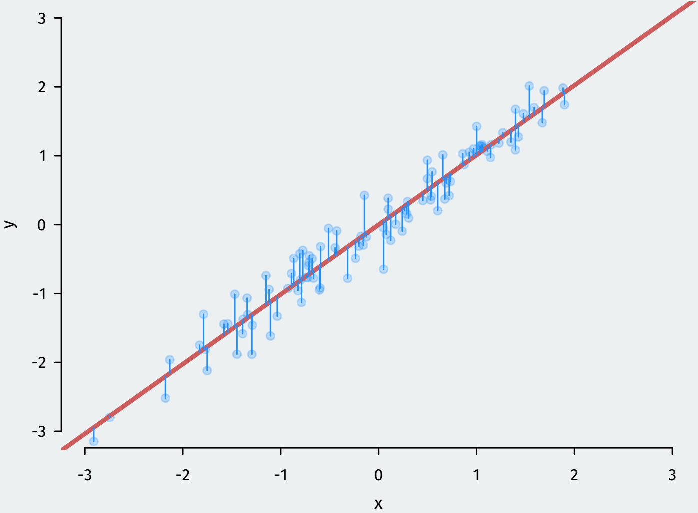
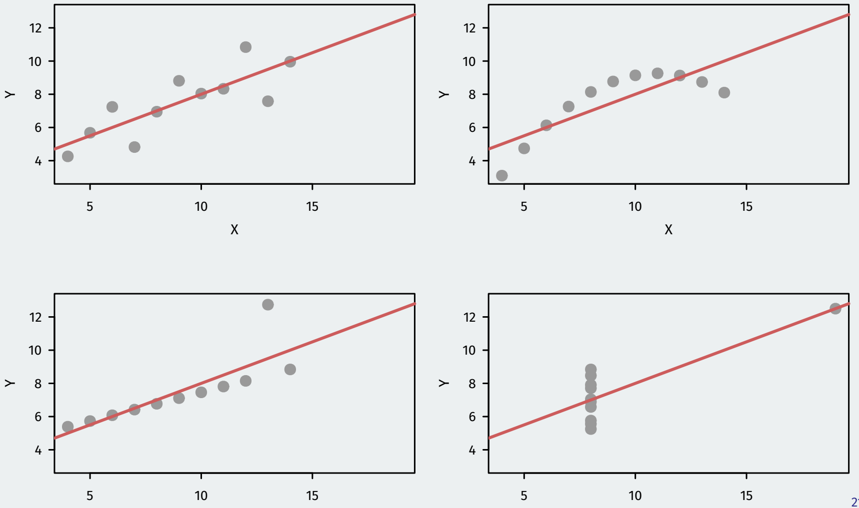


Is R-squared useful?

- Can be very misleading. Each of these samples have the same R^2 even though they are vastly different:



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- Could waste tons of governmental or corporate resources with a bad prediction model!

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- Congrats, you know machine learning/artificial intelligence!

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$$\texttt{seat.change}_i = \alpha + \beta_1 \texttt{approval}_i + \beta_2 \texttt{rdi.change}_i + \epsilon_i$$

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- $\widehat{\beta}_2 =$ 4.213: average increase in seat change for each additional percentage point increase of RDI, **holding approval fixed**

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$$\widehat{\epsilon}_i = Y_i - \widehat{\alpha} - \widehat{\beta}_1 X_{i1} - \widehat{\beta}_2 X_{i2}$$

- Find the coefficients that minimizes the **sum of the squared residuals**:

$$\text{SSR} = \sum_{i=1}^{n} \widehat{\epsilon}_i^2 = (Y_i - \widehat{\alpha} - \widehat{\beta}_1 X_{i1} - \widehat{\beta}_2 X_{i2})^2$$

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On deck

- Next week:

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  - Allowing for different slopes for different groups of observations.

On deck

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  - Allowing for non-linear effects!